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including all sensible personality. This renunciation consists in a withdrawal of attention. This withdrawal of attention is accomplished by means of active denial of the existence of the world of sense. The attention is then centered on the revealed concept of being as found in the sacred books. The results which follow from this turning from the sensible to the supersensible form a body of facts which demonstrate the truth of revelation. These facts are accessible to all who choose to make the experiment of denying the world and recognizing the All-one. Thus through religious action there is opened an unlimited field of practical intercourse with the supersensible. And through this intercourse an intimate practical acquaintance with God may be attained even by those who have never entered the state of mystic experience.

We live in a dualistic world. By rising above the world the problem of the world is solved for us.

Ernst Jonson.

New York City.

GALILEO AND NEWTON.

It is perhaps advisable to consider Newton's debt to Galileo both in mechanics and mathematics rather more fully than I have done in my previous articles on Newton and the principles of mechanics in this magazine. I will try to follow the thread connecting the thoughts of these men in what follows.

One of the most striking results of Galileo's *Discorsi,* published in 1638, is that a motion does not need a force to keep it up. This fact is concealed by the constant presence around us of friction and resistance, and so even Descartes had imagined that each planet is kept in motion by a vortex in a fluid which fills all space. Galileo found, in fact, that a force changes the *velocity* of a body. Thus, without a force which would deflect a planet into a curved orbit, the planet would proceed with uniform velocity along a straight line. Also, when considering the paths of projectiles, Galileo showed that the paths arise from compounding a horizontal uniform velocity and a vertical accelerated motion of falling. The resultant of two motions at right angles to one another is a motion along the diagonal of a parallelogram whose adjacent sides are along the two directions of motion. By considering the various points of arrival for horizontal velocity and vertical acceleration of fixed magnitudes but for various periods of duration, Galileo found that all the points lie on a parabola.

Both these advances made by Galileo were apparently quite familiar to such physicists as Huygens, Wren, Hooke, and Halley. Huygens made a very important application of Galileo's ideas to motion in a circle round a force at the center of the circle. This is, for example, the case of a stone placed on a smooth horizontal table, so as to neutralize the effect of the stone's weight, and then whirled round at the end of a string of which the other end is kept still. In this case the stone is continually deflected from a tangential path by a pull along the string to the center of the circle. We know that Newton discovered quite independently in 1666 the main result of Huygens by a method which was very probably rather different from that of which Huygens published some results in 1673. Huygens noticed that, if a circle of radius r is described with uniform velocity v, any new acceleration must, since there is no change of velocity in the orbit of the body, be at right angles to the circle at the point where the moving body is. There is thus a continual acceleration due to the force acting toward the center of which the "centrifugal force" along the tangent is the accompanying "reaction," as Newton afterward called it, and this acceleration is used up in continually changing the direction of the velocity without bringing the body any nearer to the center of the circle. If no force acted between the center and the body, the body would proceed with uniform velocity v in a straight line PT touching the circle at P, and in a time t would travel a distance vt along this line. Again, if the body at P had no velocity along PT, it would, in the time t, by Galileo's formula, travel a distance $at^2/2$ toward the center, where a is the acceleration. Of course in this case we cannot imagine the force toward the center to be transmitted by a perfectly inextensible string, for unless the string were kept stretched or the time t made infinitesimally short, there would be no force along the string. The actual motion of the body is compounded of both these motions taking place at the same time, and, if we consider t as infinitesimal, the resultant path may be considered as a straight line PQ which coincides with an arc of the circle, so that Q is a point on the circle infinitely near P. The distance TQ, which is parallel to the line joining P to the center (S) represents the distance fallen through toward the center in the short time t. Since P and Q are on a circle, the lengths PT (y), TQ (x) are connected by the relation

$$y^2 + (r-x)^2 = r^2$$

or, what is the same thing, by the relation between y^2 and $x(2r-x)$ proved in Euclid's *Elements* (III, 36). The above relation gives, when we substitute for y and x in it from what precedes and neglect powers of t higher than the lowest,

$$v^2 = ar.$$

From this we get Huygens's formula for the magnitude of the central force,

$$a = v^2/r.$$

It may be mentioned in passing that there is some analogy between this investigation and Galileo's investigation of the path of a projectile. If the projectile starts from P with a velocity v along PT and travels a distance vt in the time t, and at the same time suffers an acceleration in the direction PS by which in the time t it describes a space $gt^2/2$, it arrives, by the end of the time t, at the point Q, where

$$y^2 = v^2t^2 = 2v^2x/g.$$

The difference lies in the fact that in this case t need not be infinitesimal. We may say that, in the former case, the curve *begins* by being a parabola, but that the direction of the force only remains parallel to its original direction at an infinitely small distance along the curve from P, where begins the motion that we are considering.

When the empirical laws of Kepler became generally known, it seems that they were combined with Huygens's theorem by Halley and possibly others as well as by Newton, and this combination gave rise to speculation on the orbits of the planets. Indeed, it seems natural to consider Galileo's parabola of projection as passing over into an ellipse or a circle when the center of attraction is brought from infinity to a finite distance. It is probable that Hooke had no more grounds for his assertions that the force of gravitation varies inversely as the square of the distance and that this law proves that the planets move in ellipses, than this plausible analogy together with the discovery—made also by Newton in 1666 and Halley at a later date—that Kepler's third law and Huygens's theorem between them imply that the force keeping a body revolving in a circle about a larger one, as is approximately the case with the planets and the sun or the moon and the earth, is inversely as the square of the distance.

Another aspect of Galileo's works was of especial importance in influencing Newton probably through Barrow, and this was in a purely mathematical direction. Galileo, in his attempt to find the law according to which a body falls near the surface of the earth, neglecting the resistance of the air, assumed that the velocity acquired by the end of a certain time during which the body falls is proportional to the length of the time of falling. This assumption turned out to be correct, and a previous mistaken assumption that the velocity acquired is proportional to the *space* fallen through will be referred to below. Since it was easier to find out by experiment in what way the *distance* fallen through increased with the time rather than in what way the *velocity* increased with the time, Galileo deduced, from the assumption that v is proportional to t, the relation between s and t. It must be remembered that Galileo was perfectly familiar with the ideas which were expressed in the methods of indivisibles of Kepler and Cavalieri. Galileo considered, unlike his predecessors, velocities varying from point to point, and consequently saw that we could not define "*the* velocity at a point" by the ratio of the space passed over in a finite time to that time, for different lengths of time would give different results. When, however, we consider, round a certain point, a distance which is infinitely small and therefore very nearly a straight line, then for the infinitely small time in which this space is described, we may regard the increase or decrease of space as uniform. This new notion of "velocity" as the (unique) ratio of infinitesimals includes the old one as a particular case; for if the ratio of ds to dt, as we may write these infinitesimal increments in the notation subsequently introduced by Leibniz, is constant, then s is proportional to t.

Galileo represented the lengths of time by lengths on an axis of abscissæ measured from a fixed point on it, and the magnitudes of the corresponding velocities by ordinates. In this diagram, which is like the diagrams introduced into geometry by Descartes soon after Galileo's ideas were formed, except that x and y replaced t and v, Galileo's assumed proportionality of v to t is represented by a straight line through O. That is to say, if P is any point on the t-axis and PQ is the corresponding ordinate at right angles to OP, then, when P varies in position, the ends Q of all such ordinates lie on the above straight line. Now Galileo proved that the triangular area OPQ represents, in units of square measure, the space (s), in units of linear measure, fallen through. Just as the velocity when variable can be measured, at any point, by ds/dt, the slope of the curve at that point, on a diagram in which times are abscissæ and spaces ordinates, so on a diagram in which times are abscissæ and velocities ordinates, the acceleration is measured by dv/dt, the slope of the curve. It can hardly be doubted that Galileo, knowing how areas of curves are found by the method of indivisibles, saw that when the acceleration is *variable,* the area of the figure OPQ, which is no longer a triangle, still represents s. It is also quite possible that Galileo saw in this way the inverse relation of the problems of tangents and quadratures: the ordinate (v) of any point on the st-diagram is given by calculating the corresponding area on the vt-diagram, while the ordinates on the vt-diagram are determined by the slopes or tangents at the corresponding points of the st-curve. Here we may mention that, in Galileo's mistaken assumption referred to above of the proportionality of v to s, triangles like OPQ on a vs-diagram do not represent s. It will be remembered from Mach's *Mechanics* that Galileo rejected this mistaken assumption on grounds which were also mistaken. In fact, if $v=ds/dt$, the integral of $v.dt$ is s, but the integral of $v.ds$ is not s unless v is always unity, so that a vs-diagram does not show that integration is the inverse of differentiation. The notions of time being the independent variable and geometrical curves being generated by motions were used by Isaac Barrow, who was certainly influenced by Galileo and possibly by Roberval. Barrow denoted the areas by "t" and "v," and in this notation and the ideas which it implied he was followed by his pupil Isaac Newton, who, in his "method of fluxions," greatly developed the suggestive ideas of Barrow, especially Barrow's clear perception of the truth that the problems of tangents and of quadratures were inverse problems.

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CURRENT PERIODICALS.

We hope that the increasing bulk of *Science Progress,* this "Quarterly Review of Scientific Thought, Work and Affairs," is the outward and visible sign of an increasing recognition of its interest